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ELECTRIC ACTION OF A FLAME AND A CONDUCTING LIQUID JET IN AN ELECTRIC FIELD

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Expressions are obtained for the charge and potential of a spheroid-shaped burner with the flame on its top situated in an external field parallel to its rotation axis. Simple expressions for potentials of the burner are obtained in terms of the external field for the sphere, strongly oblate, and strongly stretched ellipsoids. In an analogous manner, electrification of a spherical tank with the outflowing conducting liquid is considered, and the charges and potentials of the field strength gauge are determined.

W. Thomson (Lord Kelvin) discovered that a tank with an outflowing water jet or with a rising flame acquires a charge in an electric field, and used this effect in his studies of the terrestrial electric field [1]. In the case of a spherical tank with a small flame on top, calculations of the equilibrium charge value are elementary [2] and can also be carried out for an oblate or stretched spheroid-shaped tank, and in the case of limiting configurations the results are obtained by exceptionally simple expressions. Here we consider the process of charge equilibration on such a tank situated in a field E directed along the axis of rotation (the opening with the flame is situated at a top of the tank, and the opening has small dimensions compared to semiaxes a and b) (Fig. 1).

The charge density at the top σ originates from two phenomena: if the tank has acquired a charge Q sign of opposite to that carried away by the flame, then, according to the results of calculations [3]

$$\sigma^{\mathrm{I}} = \frac{Q}{4\pi b^2} \tag{1}$$

and, in addition, an induced charge is created in the external field with the density at the top equal, according to [3], to

$$\sigma^{\mathrm{II}} = \frac{\varepsilon_b E}{n_x}, \tag{2}$$

where n_x is determined by the ratio of the semiaxes. The strength of the current created by the air flow, which rises at a rate u with the charge density on its surface assumed to be equal to $\sigma^{\mathrm{I}}+\sigma^{\mathrm{II}}$, is written as

$$I = 2\pi r\,(\sigma^{\mathrm{I}} + \sigma^{\mathrm{II}})\,u\,. \tag{3}$$

If one takes into account the charge loss due to the scattering in the air [4], which is characterized by the relaxation time τ_s, then the variation of the tank's charge with time is determined from the equation

$$\frac{dQ}{dt} = -I - \frac{Q}{\tau_r} = -\left(\frac{ru}{2b^2} + \frac{1}{\tau_r}\right) Q - \frac{2\pi ru\varepsilon_b}{n_x} E\,. \tag{4}$$

The charge relaxation time is determined by the following expression:

$$\frac{1}{\tau} = \frac{ru}{2b^2} + \frac{1}{\tau_r}, \tag{5}$$

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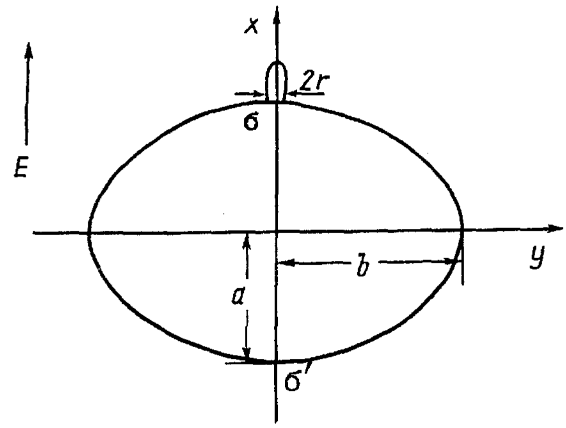

Fig. 1. Ellipsoid-shaped burner.

and the equilibrium charge value is

$$Q_\infty = -\frac{4\pi r u b^2 \tau_r \varepsilon_b}{n_x (2b^2 + r u \tau_r)} E. \tag{6}$$

If the scattering time is long enough, so that the second term in (5) can be neglected, then the charge value is independent of the convection rate:

$$Q_\infty = -\frac{4\pi b^2 \varepsilon_b}{n_x} E. \tag{7}$$

The corresponding value of the potential is also determined by the capacity C:

$$V_\infty = -\frac{4\pi b^2 \varepsilon_b}{C n_x} E. \tag{8}$$

The equilibrium regime starts when the charge density at the flame's origin vanishes and the charge is no longer carried away. In this case the charge density at the opposite bottom takes on the value

$$\sigma' = -\frac{2\varepsilon_b E}{n_x}. \tag{9}$$

Let us consider the limiting types of the tank's shape for which the expressions for the potential are simplified substantially [3].

In the case of a spherical shape, $C = 4\pi\varepsilon_b b$, $n_x = 1/3$, and

$$V_\infty = -3bE, \tag{10}$$

where b is the radius of the sphere.

In the case of a strongly oblate ellipsoid, $C = 8\varepsilon_b b$, $n_x = 1$, and

$$V_\infty = -\frac{\pi}{2} bE, \tag{11}$$

where b is the semimajor axis.

In the case of a strongly stretched ellipsoid, the asymptotic characteristics are as follows:

$$C = \frac{4\pi\varepsilon_b a}{\ln\frac{a}{b}}, \quad n_x = \frac{b^2}{a^2}\ln\frac{a}{b}$$

and

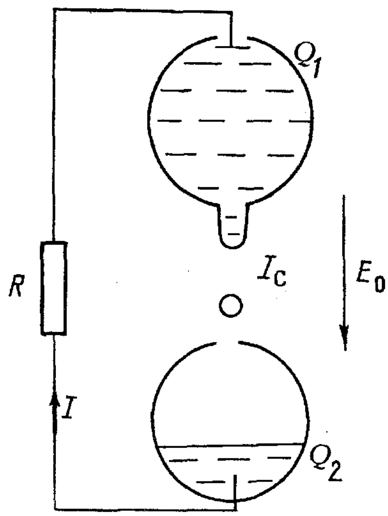

Fig. 2. Electric field strength gauge.

$$V_\infty = - aE, \tag{12}$$

where a is the semimajor axis. The charge density (9) can take high values in this case.

An analogous analysis is carried out for the electrification of a tank taking place as a result of the outflow of a conducting liquid whose jet breaks up into droplets. Let us carry out the corresponding calculations assuming that the jet can have a length comparable to the dimensions of the tank. We represent the jet as a strongly stretched semispheroid connected to the tank which is assumed to have a spherical shape, and we direct the polarizing field along the axis of the semispheroid. According to [5], the image of the charged semispheroid created in the infinite conducting plane completes the semispheroid, so that the polarization of the spheroid as a whole can be considered. The induced charge density in an arbitrary point is written as [3]

$$\sigma = \frac{\varepsilon_b E}{n_x} \nu_x, \tag{13}$$

where ν_x is the projection of the unit vector normal to the surface onto the rotation axis.

Denoting the semimajor axis α, and the semiminor axis β, with $\beta << \alpha$, one can represent the charge of a newly created droplet as the product of half its surface $2\pi\beta^2$ by the charge density at the top. More accurate calculations taking into account the decrease in density with distance from the top, which we omit here, shows that the result should be halfed, so that the following expression is assumed for the droplet charge:

$$q = \frac{\pi\varepsilon_b\alpha^2}{\ln\frac{\alpha}{\beta}} E. \tag{14}$$

If N droplets are detached per second, then the convection current strength is as follows:

$$I_c = \frac{\pi\varepsilon_b\alpha^2}{\ln\frac{\alpha}{\beta}} NE. \tag{15}$$

The polarizing field is made up of the external field E_0 and the field of the tank's charge which is assumed to be homogeneous within the region of the jet. For a spherical tank of radius r_t carrying the charge Q_1 the polarizing field is expressed as $E = E_0 + Q_1/4\pi\varepsilon_b r_t^2$, and the convection current strength is

$$I_c = \frac{\pi\varepsilon_b\alpha^2}{\ln\frac{\alpha}{\beta}} NE_0 + \frac{\alpha^2 N}{4r_t^2 \ln\frac{\alpha}{\beta}} Q_1, \tag{16}$$

where the coefficients at E_0 and Q_1 will be referred to in what follows as A and B, correspondingly. Here we assume that the external field acts on the jet but does not act on the tank (the charge is brought to the jet); if the tank is situated in the field, it is polarized and the external field (16) is tripled. In the terrestrial field strength gauge [1] water flows out from the tank onto the ground, and the equilibrium state in which the convection current vanishes is considered. The corresponding tank's charge is determined from (16) subject to the condition $I_c = 0$. In so doing, it is seen that the tank's grounding leads to the appearance of a permanent convection current, since the charge is carried away, setting thus the device to the initial state. In other experiments water flows out of the upper tank into the lower one, and the latter acquires a charge sign opposite to that of the upper tank.

Let us consider the action of the device in which the tanks are connected via the resistance R. Assuming R is equal to zero or infinity, we have the case of short-circuited or disconnected tanks (Fig. 2). We denote the current via the resistance by I, and the charges and capacities of the upper and lower tanks by Q_1, C_1, and Q_2, C_2, respectively. The equations for the charges and densities in the device are as follows:

$$Q_1 + Q_2 = 0\,; \quad \frac{dQ_1}{dt} = I - I_c\,; \quad \frac{Q_2}{C_2} - \frac{Q_1}{C_1} = IR\,. \tag{17}$$

The solution for the stationary regime can be found from (16) and (17):

$$Q_{1\infty} = -\frac{RC_1C_2AE_0}{C_1 + C_2 + C_1C_2BR}, \tag{18}$$

$$I_{c\infty} = \frac{A\,(C_1 + C_2)\,E_0}{C_1 + C_2 + C_1C_2BR}. \tag{19}$$

At $R = 0$ we obtain $Q_{1\infty} = 0$, $I_{c\infty} = -AE_0$, whereas at $R = \infty$ we obtain $Q_{1\infty} = -(A/B)E_0$, $I_c = 0$. In the latter case the tank's field neutralizes the external field, and the charging of droplets stops.

In the nonstationary regime the charge is determined by the equation

$$\frac{dQ_1}{dt} + \left(\frac{C_1 + C_2}{RC_1C_2} + B\right) Q_1 = -AE_0\,. \tag{20}$$

The relaxation time of the process equals to

$$\tau = \frac{RC_1C_2}{C_1 + C_2 + RC_1C_2B}. \tag{21}$$

At $R = 0$ we obtain $\tau = 0$, whereas at $R = \infty$ we obtain

$$\tau = \frac{1}{B} = \frac{4r_t^2 \ln\frac{\alpha}{\beta}}{\alpha^2 N}. \tag{22}$$

A numerical estimate with $r_c = 0.2$ m, $\alpha = 0.05$ m, and $\beta = 0.001$ m, $N = 1\ \mathrm{sec}^{-1}$ shows that the relaxation time is about 3 min.

If the droplets fall out immediately from the tank situated in the field E_0, then, at a charge density at the top $\delta = 3\varepsilon_b E_0$, opening radius r, and mean outflowing rate u, the convection current strength is equal to $6\pi\varepsilon_b r E_0 u$. The mean outflowing rate is determined by the relationship $\pi\beta^2 u = (4\pi/3)\beta^3 N$, so that the current strength $I_c = 8\pi\varepsilon_b\beta^2 N E_0$. Comparison with Eq. (16) shows that the jet enhances the current by approximately $(\alpha/\beta)^2$ times.

Although field gauses based on flame and jet electrification have not been in use for decades, the effects themselves have not lost their importance. They manifest themselves in nature (volcanos, geysers) and in technical devices (thermal engines, fountains).

NOTATION

E, field strength; a, b, α, and β, semiaxes of the ellipsoid; δ, surface charge density; Q_1 and Q_2, charges of the tanks; g, charge of the droplet; n_x, shape coefficient; I, current strength; C, capacity; R, resistance; r, radius; τ, relaxation time; u, flow rate.

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